

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA16228

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Details of the STSM

Title: Client-Waiter games

Start and end date: 26/03/2022 to 02/04/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

We were looking at the unbiased positional game of the Waiter-Client type played on the edges of a given graph, for several well-studied graph structures like a positive minimum degree, perfect matching or a Hamiltonian cycle. We wanted to obtain bounds for the asymptotic minimum density of the board graph on which Waiter can win. In particular, we were hoping to relate the behaviour of the Waiter-Client games to the games of the Maker-Breaker type. As previous bounds were far from each other, we aimed at gaining a better understanding of this central and long standing open problem.

We also studied the game played on the edges of a given graph in which the first player's goal is to claim a majority of edges on as many vertices as possible, with particular attention devoted to trees.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

The new upper bound on the asymptotic minimum density on which Waiter can still win was obtained for Hamiltonicity game in the Waiter-Client setting. This is a polynomial improvement on the previously best known bound. As for the lower bound, we were able to adopt approaches used for answering the same question in Maker-Breaker games. We are confident that a similar construction can be tailored to fit the same game in the biased setting.

Some results and insights were also obtained in the Perfect Matching game in relation to the positive minimum degree game, providing a lower bound. On the other hand, we have some constructions that

¹This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

give an upper bound. In the minimum degree k game we could construct a sparse small graph on which Waiter can win the game, which is a good base for a general construction.

Finally, we looked at the game played on the edges of a given graph in which the first player's goal is to claim a majority of edges on as many vertices as possible. Special attention was devoted to the case when the base graph is a tree, where we have determined the leading term of the worst-case number of vertices the first player can dominate. We established relations between this game and the so-called Isolation game, where the goal of the first player is to touch as many vertices as possible.

For several following weeks, we will continue trying to improve some of the above mentioned results. Consequently, we will proceed to writing a paper, which we plan on submitting to a leading scientific journal.

During my visit to Utrecht University, we discussed several possibilities for future collaboration, both for further efforts to attack the above mentioned problems, as well as for other forms of teamwork.