Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA16228

Grantee name: Dr. Michael Mc Gettrick

Details of the STSM

Title: Evolutionary quantum games on graphs

Start and end date: 09/01/2022 to 22/01/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

Under this STSM, Dr. Mc Gettrick (NUI Galway) visited Prof. Siegmund (TU Dresden) for two weeks, to work on evolutionary quantum games on graphs. Dr. Jeremias Epperlein (Universität Passau) also joined the collaboration. Initial discussions were on broad definitions, mainly concerned with how to define the link between the "game" and the "graph". It was quickly realised that, even for 2-player case (where each node of the graph represents a player and each edge a game) there are features of the quantum game that are hard to define using the graph, specifically the representation of the initial entanglement (from which the quantum game derives its advantage). In other words, a given graph shows "who plays with who", but does not seem to represent completely the different types of entanglements possible in multipartite quantum states.

The research moved in two slightly different directions: One was, to try to represent diagrammatically using some additional structure (a multigraph) the entanglement. Mutligraphs have properties that seem certainly to make them a stronger candidate for representing multipartite entanglement. Some of these properties were explored (for example for tripartite GHZ and W states), though it is not clear yet how to define this representation in the most general cases.

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

The second avenue, on which more time was spent, was looking at the 2-player iterated CHSH game. After each round of the game, it is clear the quantum state has zero entanglement (because of measurement), so a model was built with nearest-neighbour quantum Ising Hamiltonian (the so-called transverse-field Ising model), which is "turned on" between each game to rebuild the entanglement for the subsequent game. We calculated, based on parameters in the Hamiltonian, how long we need to maintain this interaction to produce maximum entanglement, and hence the optimum quantum winning probability (about 0.86) for each iterated quantum game.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

During the STSM, we already started to write up our calculations and results, for the iterated CHSH game with a transverse-field Ising model interaction. We intend to publish this work soon in a research journal, and this will be one of our main outcomes. I would certainly say the STSM achieved its goals. This is a new field, with many open problems, and many new concepts. We are continuing work (with Jeremias Epperlein) in the direction of trying to find an appropriate (graph?) structure to encode general entanglements. A new mini-team has been extablished, with common interests, eash contributing from slightly different backgrounds to the research (Mc Gettrick from (theoretical) physics, Siegmund from mathematics, Epperlein from computing). We continue our work using ZOOM calls, and further research collaborations are scheduled (with Siegmund coming to NUI Galway in September 2022).

In this action, a main objective is that "computer scientists, applied mathematicians, economists, and operations researchers will join forces" (https://gametheorynetwork.com/action/about-game-theory-network/) and we certainly have done this in this STSM.