Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA16228

Grantee name: Fryderyk Falniowski

Details of the STSM

Title: Game dynamics and chaos

Start and end date: 20/11/2021 to 28/11/2021

Description of the work carried out during the STSM

Motivated by recent results showing chaotic behavior for Follow the Regularized Leader algorithm (FTRL) with steep regularizers in congestion games, resulting in large regret even in games with optimal Price of Anarchy, we studied whether the chaotic phenomenon is strictly connected with the type of algorithms or is it present for other discrete dynamical systems introduced by learning in games.

We compared properties of dynamical systems arising from congestion games in discrete versions of three models, which in continuous time give replicator dynamics:

1. Discretization of Taylor-Jonker biological replicator dynamics,
2. Multiplicative Weights Update (exponential variant),
3. Discretization of Pairwise Proportional Imitation protocol dynamics.

Although in continuous time all these dynamics are equivalent, in the discrete case this is no longer the case. Surprisingly, even for a simple congestion game the dynamics give qualitatively different long-term behavior. Taylor-Jonker approach results convergence while for MWU and PPI one can see instability and chaotic behavior (in the Li-Yorke and positive topological entropy sense). Thus, the system can become unpredictable. Our work shows that the chaotic behavior is not a side effect of using the FTRL algorithm in congestion games but can be seen for other algorithms.

Description of the STSM main achievements and planned follow-up activities

The aim of this STSM was to study whether phenomenon of chaotic behavior observed for FTRL algorithms in congestion games is strictly connected with the type of algorithms. We showed that in fact this is more general effect. We studied discrete dynamical systems arising from different variants of

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

replicator dynamics in congestion games. In particular we showed chaotic behavior for discrete time version of pairwise proportional imitation protocol.

This rise a list of questions to follow: how chaotic behavior arise in games? What are conditions for games where chaotic behavior can be seen? What are conditions for algorithms? How complex the system can become? What about other properties connected usually with chaos?

We also plan to study other problems in game theory and optimization, e.g. limitations of gradient-descent algorithms, through the lenses of discrete dynamical systems.

These and other questions will be studied in the future in collaboration initialized by this STSM.

Finally, this STSM has served an objective of GAMENET to describe dynamics of algorithms for effective control of multi-agent optimization problems.