

Coalition-Proof Risk Sharing Under Frictions

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Introduction

- All institutions (including Pareto-improving ones) are essentially self-enforcing agreements (based on a shared belief in future cooperation).
- A belief in the possibility of future cooperation can also encourage coalitional deviations, undermining the institution.
- But such belief may be tenuous and difficult to achieve. We refer to the ease with a community can achieve a belief in future cooperation as its **social capital**.
- We investigate the role of social capital in the endogenous formation of self-enforcing insurance arrangements.



Related Literature

- ① Coalition- and renegotiation-proofness: von Neumann and Morgenstern (1944), Greenberg (1990), Farrell and Maskin (1989), Bernheim, Peleg, and Whinston (1987), Bernheim and Ray (1989)
 - A putative agreement is destabilized if after some history there is dominating agreement available to some or all the players. Assumes dominating agreement will result if present.



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 - A putative agreement is destabilized if after some history there is dominating agreement available to some or all the players. Assumes dominating agreement will result if present.
- 2 Risk sharing with limited contract enforcement: many contributions, closest is Krueger and Perri (2011)
- 3 Risk sharing with endogenous outside option: Fewer contributions, closest is Krueger and Uhlig (2006)
- 4 Risk sharing with coalition deviations: Genicot and Ray (2003)



Main Results

As social capital increases from no social capital (so no agreement):

- 1 Initially, the ex ante value of insurance is strictly increasing, with first best insurance being implemented if agreement when reached (which is unlikely, so coalitional deviations are not a concern).



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- 1 Initially, the ex ante value of insurance is strictly increasing, with first best insurance being implemented if agreement when reached (which is unlikely, so coalitional deviations are not a concern).
- 2 For higher social capital, coalitional deviations are a binding concern. Equilibrium insurance is imperfect and nonstationary (poor agents' consumption is decreasing in the time they have been poor). The ex ante value of this imperfect insurance is strictly increasing.
- 3 For highest social capital, the threat of coalitional deviations may imply that the ex ante value of equilibrium insurance is constant with the insurance requiring increasing **utility burning** as social capital increases.



Classic Insurance Economy

- Continuum population of ex ante identical risk averse agents subject to income risk.
- Each individual has low income ℓ and high income $h > \ell$ with equal probability. Set $Y := \{\ell, h\}$, and $\bar{y} := \frac{1}{2}(\ell + h)$. Individuals cannot store.
- Large (i.e., positive measure) coalitions have no aggregate risk.
- Individual's utility function is a strictly concave u , and she maximizes the average expected discounted sum of utility, discount factor β .
- Autarkic utility:

$$V^A = \frac{1}{2}(u(\ell) + u(h)).$$



Coalition Formation and Deviation

- Initial period, $t = 0$, is the planning/contracting period.
- **Social capital** measured by π : Coalition only forms with prob $\pi \in [0, 1]$.
- If initial coalition does not form, then in autarky forever.
- If a coalition does form, then it agrees to a **social norm** of time paths of consumption, and starting in the next period:
 - Attempt to form new coalition (deviation) requires exit from old coalition.
 - Attempt succeeds with probability π .
 - If attempt fails, then autarky forever after.
- Any agreement reached is done cognizant of the possibility that a (further) subset of agents may secede.



Allocations

- A **consumption allocation** is $c : \cup_{t=1}^{\infty} Y^t \rightarrow \mathbb{R}_+$.
- **Ex ante utility** from an arbitrary allocation c is

$$W^0(c) := (1 - \beta) \sum_{\tau=1}^{\infty} \sum_{y^\tau} \beta^{\tau-1} \Pr(y^\tau) u(c(y^\tau)).$$

- **Continuation utility** from c at a history $y^t \in Y^t$ is

$$W(y^t, c) := (1 - \beta) \left\{ u(c(y^t)) + \sum_{\tau \geq 1} \sum_{y^\tau} \Pr(y^\tau) \beta^\tau u(c(y^t y^\tau)) \right\}.$$

- **Secession utility** from c at history $y^t = (y_1, \dots, y_t)$ is

$$(1 - \beta)u(y_t) + \beta[\pi W^0(c) + (1 - \pi)V^A].$$



Equilibrium

Definition

An allocation is an **equilibrium** if it solves

$$\max_c W^0(c)$$

subject to

- 1 **resource feasibility**: $\sum_{y^t} c(y^t) \Pr(y^t) = \bar{y}$ for all $t \geq 1$, and
- 2 **internal-incentive feasibility**: for all $t \geq 1$ $y^t \in Y^t$,

$$W(y^t, c) \geq (1 - \beta)u(y_t) + \beta[\pi W^0(c) + (1 - \pi)V^A].$$



Complete Insurance

- Ex ante utility with complete insurance is

$$V^{FB} := u(\bar{y}).$$

- For complete insurance to be an equilibrium, need

$$V^{FB} \geq (1 - \beta)u(h) + \beta(\pi V^{FB} + (1 - \pi)V^A).$$

- First best fails to be an equilibrium when π is too large.
- At $\pi = \pi^{FB}(\beta) < 1$, h -incentive feasibility is **just** binding, while l -incentive feasibility is slack.
- Even $\pi = 0$ is too large if β is too small ($\beta < \beta^{FB}$, where $\pi^{FB}(\beta^{FB}) = 0$).



Equilibrium (Possibly Incomplete) Insurance

Definition

An allocation c is an **equilibrium** if c maximizes $W^0(c)$ subject to resource feasibility and for all $t \geq 1$ $y^t \in Y^t$,

$$W(y^t, c) \geq (1 - \beta)u(y_t) + \beta[\pi W^0(c) + (1 - \pi)V^A].$$

- Existence is immediate, characterization is not.
- The program defining equilibrium allocations is not concave (in particular, the constraint correspondence is not convex valued).
- We follow an indirect route, via fixed points of a mapping $T(\cdot; \pi)$.



Exogenous Incentive Feasibility

Fix $F \in \mathbb{R}_+$. Maximize $W^0(c)$ over resource feasible allocations c satisfying **F -incentive feasibility**: for all $t \geq 1$ $y^t \in Y^t$,

$$W(y^t, c) \geq (1 - \beta)u(y_t) + \beta F.$$

Denote the set of feasible c by $\mathcal{C}(F)$ and the value of this program $V^*(F)$.



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If c^\dagger is an equilibrium allocation, then

$$W(y^t, c^\dagger) \geq (1 - \beta)u(y_t) + \beta[\pi W^0(c^\dagger) + (1 - \pi)V^A],$$

and so so $c^\dagger \in \mathcal{C}(F^\dagger)$ for $F^\dagger = \pi W^0(c^\dagger) + (1 - \pi)V^A$.



Exogenous Incentive Feasibility

Fix $F \in \mathbb{R}_+$. Maximize $W^0(c)$ over resource feasible allocations c satisfying **F -incentive feasibility**: for all $t \geq 1$ $y^t \in Y^t$,

$$W(y^t, c) \geq (1 - \beta)u(y_t) + \beta F.$$

Denote the set of feasible c by $\mathcal{C}(F)$ and the value of this program $V^*(F)$.

Theorem

Suppose F is a fixed point of

$$T(F; \pi) := \pi V^*(F) + (1 - \pi)V^A,$$

and $c^\dagger \in \mathcal{C}(F)$ is the argmax, i.e., $V^(F) = W^0(c^\dagger)$. Then F is the only fixed point and c^\dagger is the unique eq allocation and F is its **ex ante** value.*



The Fixed Point

- Suppose c^\dagger is the eq allocation associated with the fixed point F .
- Then c^\dagger is a self-enforcing social norm in a **strong** sense: it survives the threat of secession by any coalition that reoptimizes over all possible **coalition-proof** allocations (this is **external-incentive feasibility**, which is stronger than internal-incentive feasibility).
- Fixed point exists as long as the constraint set is nonempty for F in the “relevant” region, which may rule out large π .



Properties of $\mathcal{C}(F)$

- Observe that $\mathcal{C}(V^A) \neq \emptyset$.
- Define $\bar{F} := \sup\{F \mid \mathcal{C}(F) \neq \emptyset\}$.
- If $\beta u'(\ell) < u'(h)$ (no incentive to smooth income), then $\bar{F} = V^A$.



Properties of $\mathcal{C}(F)$

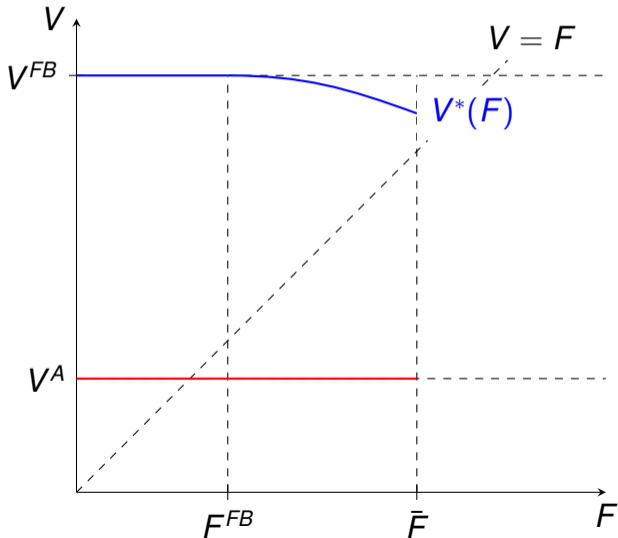
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- If $\beta u'(\ell) < u'(h)$ (no incentive to smooth income), then $\bar{F} = V^A$.
- If $\beta u'(\ell) > u'(h)$, then, $\bar{F} > V^A$: The **simple ladder** allocation

$$c(y^t) = \begin{cases} h - \varepsilon, & y_t = h, \\ l + 2\varepsilon, & y_{t-1} = h, y_t = l, \\ l, & y_{t-1} = y_t = l, \\ l + \varepsilon, & y_t = l, t = 1. \end{cases}$$

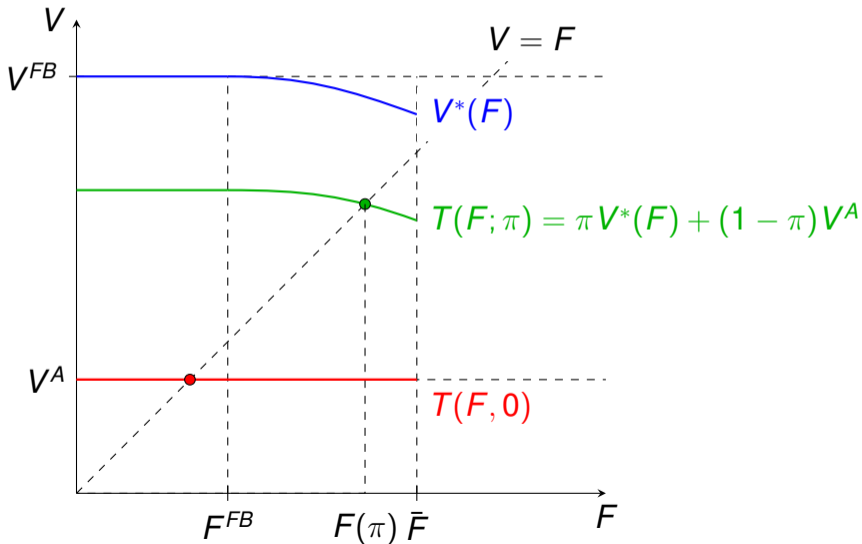
is strictly exogenous incentive feasible at $F = V^A$ (for $\varepsilon > 0$ small).
(Memoryless allocations do not work: need larger lower bound on β .)



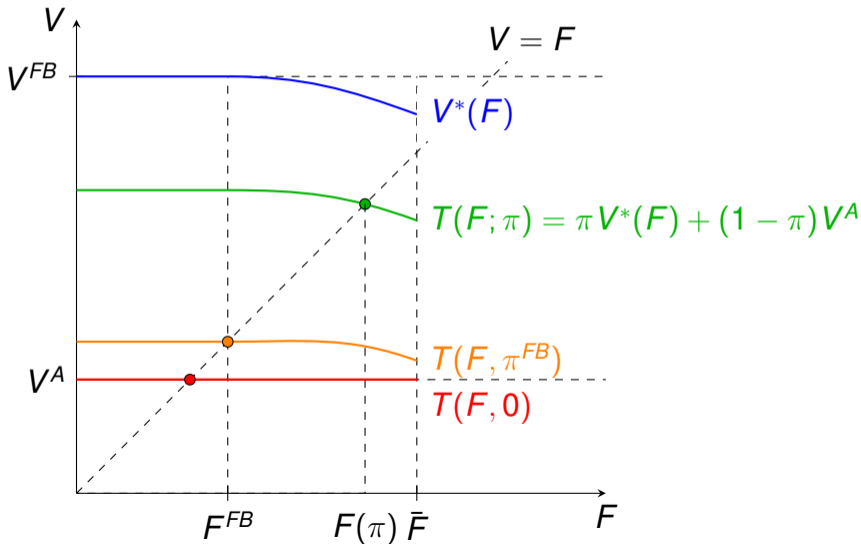
Finding the Fixed Point (fixed $\beta > \beta^{FB}$)



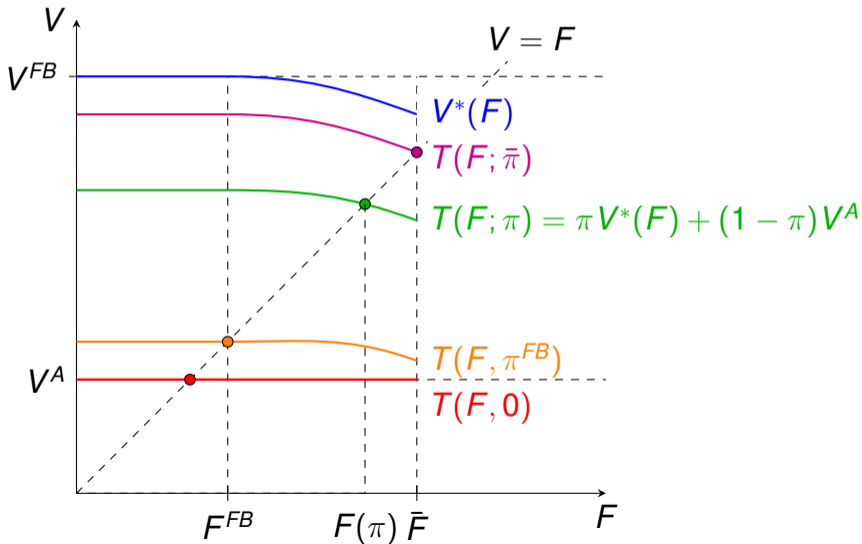
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Theorem

Equilibrium exists for all $\pi \in [0, 1]$.

- 1 *If $\beta \leq u'(h)/u'(\ell)$, there is no risk sharing in equilibrium.*
- 2 *If $\beta > u'(h)/u'(\ell)$, risk sharing occurs in equilibrium. There exists $\bar{\pi} \in (0, 1]$ such that*

 - 1 *for $\pi \in [0, \bar{\pi}]$, equilibrium is unique, its ex ante value is increasing in π , equaling $\bar{F} > V^A$ at $\bar{\pi}$, and*
 - 2 *for $\pi \in (\bar{\pi}, 1]$, equilibrium allocations are not unique, but all have the same ex ante value of \bar{F} .*

- 3 *For $\beta > u'(h)/u'(\ell)$ but not too large, $\bar{\pi} < 1$.*

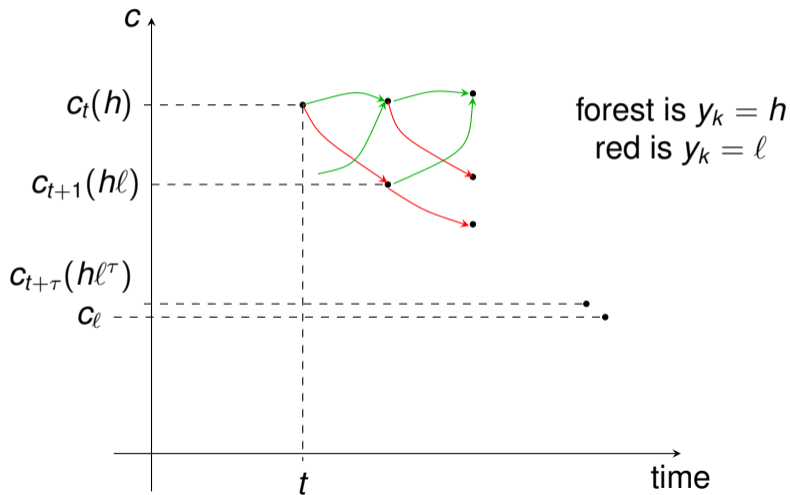


Characterizing equilibrium allocations for $\pi^{FB} < \pi \leq \bar{\pi}$

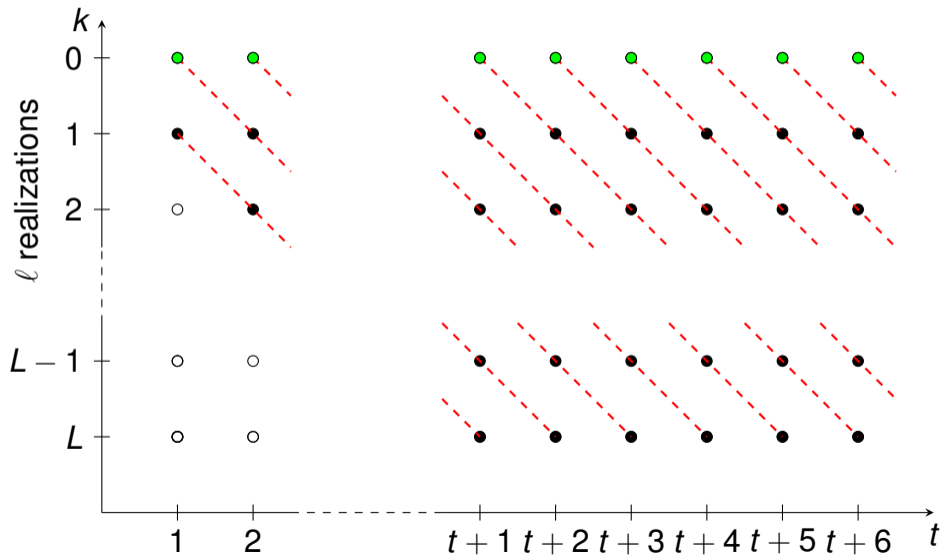
- Allocation discourages secession by h -agents by allocating insurance payments towards the more recently poor. This gives sufficiently high h -agent continuation utility with $c(y^t h) < h$ (the balance $h - c(y^t h)$ goes to current poor agents).
- l -agents' consumption is initially strictly decreasing in the time they have been poor.
- But at some point, l -agent consumption hits a floor $c_\ell > l$.
- c_ℓ is determined by l -incentive feasibility: Be seceding, an l -agent restarts the consumption ladder, which may be more attractive than being longtime poor with significant probability.



The Consumption Ladder



Ladders



What if $\pi > \bar{\pi}$?

- Recall that if c is an equilibrium allocation, then $c \in \mathcal{C}(F)$ for $F = \pi W^0(c) + (1 - \pi)V^A$.

- Since

$$\bar{F} = \bar{\pi} V^*(\bar{F}) + (1 - \bar{\pi}) V^A,$$

if $\pi > \bar{\pi}$, equilibrium requires **utility burning**:

$$W^0(c) < V^*(\bar{F}).$$

- Can be achieved in several ways. For example:

Postpone the start of risk sharing for just enough periods so that the discounted payoff is \bar{F} . This burns utility but not resources.



Return to Simple Ladder

$$c(y^t) = \begin{cases} h - \varepsilon, & y_t = h, \\ l + 2\varepsilon, & y_{t-1} = h, y_t = l, \\ l, & y_{t-1} = y_t = l, \\ l + \varepsilon, & y_t = l, t = 1. \end{cases}$$

- If $\beta > u'(h)/u'(l)$, ladder is strictly exog-incentive feasible for $F = V^A$ (equivalent to internal-incentive feasibility when $\pi = 0$):

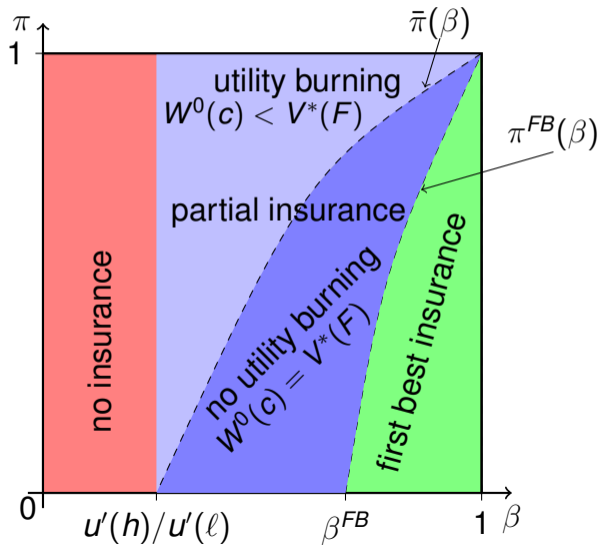
$$-u'(h)\varepsilon + (\beta/2)u'(l)2\varepsilon \geq 0.$$

- If $\pi = 1$, internal-incentive feasibility requires

$$-u'(h)\varepsilon + \frac{1}{2}\beta u'(l)2\varepsilon \geq \frac{1}{2}\beta u'(l)\varepsilon \implies \beta \geq 2u'(h)/u'(l).$$



Summarizing (position of $\bar{\pi}$ is a conjecture for large β)



Characterizing $\bar{\pi}$

At the equilibrium allocation, we have

$$W(h, c) = (1 - \beta) \sum_{k=0}^{\infty} \left(\frac{\beta}{2}\right)^k u(c_t(h\ell^k)) + \frac{\beta}{2-\beta} W^F(h),$$

where $W^F(h) := (1 - \beta)u(h) + \beta F$.

Motivated by this, we consider the following **stationary** problem

$$V^\dagger(h; F) := \max W(h, c)$$

$$\text{subject to } \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{k+1} c(h\ell^k) \leq \bar{y}$$

$$\text{and } c(h\ell^k) \geq c_\ell(F) \text{ for all } k \geq 1.$$



Theorem

$$F < (=)\bar{F} \iff V^\dagger(h; F) > (=)W^F(h).$$



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Suppose $F = \bar{F}$. The equilibrium consumption allocation converges to the unique feasible allocation \bar{c} satisfying $V^\dagger(h; \bar{F}) = W(h, \bar{c})$, that is,

$$\lim_{t \rightarrow \infty} c_t(h\ell^k) = \bar{c}(h\ell^k) \quad \forall k.$$



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- But it does not start at \bar{c} . In the CRRA case, the equilibrium allocation initially has lower consumption for h realizations (and the convergence does not occur in finite time).
- At $\bar{\pi}$, the incentive constraints on h realizations are “very” binding.



Conclusion

- 1 Low social capital societies, $\pi \leq \pi^{FB}(\beta)$.
 - There is a low probability of forming a coalition.
 - Ex ante welfare is linearly increasing in π .
 - Conditional on coalition formation, complete insurance.
- 2 Intermediate social capital societies, $\pi^{FB}(\beta) < \pi \leq \bar{\pi}(\beta)$.
 - There is a medium probability of forming a coalition.
 - Ex ante welfare is increasing in π but at a decreasing rate.
 - Conditional on coalition formation, incomplete insurance declining in π .
 - Allocations feature wasteful inequality but are intertemporally efficient.
- 3 High social capital societies, $\pi > \bar{\pi}(\beta)$.
 - There is a high probability of forming a coalition.
 - Ex ante welfare is flat in π .
 - Conditional on coalition formation, incomplete insurance declining in π .
 - Allocations feature significant inefficiencies (incl. intertemporal).



Thank you!

