Long Information Design

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Work on *zero-sum dynamic splitting games*.

Two information designers have access to independent sources of information and want to influence a decision-maker.

Outline:

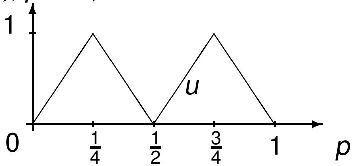
1. Introduction
2. Model
3. Games with deadline
4. Games without deadline

Introduction

Bayesian Persuasion

Some k in K is the true state. Public a priori p in $\Delta(K)$
A player chooses M and $x : K \to \Delta(M)$. Then k is chosen ac. to p, a message m is chosen ac. to $x(k)$ and publicly sent to decision-makers. Payoff to the player: $u(p(m))$, where $p(m)$ is the posterior on K given m, and $u : \Delta(K) \to \mathbb{R}$ is continuous.

Example: $K = \{0, 1\}$, $p = 1/2$.



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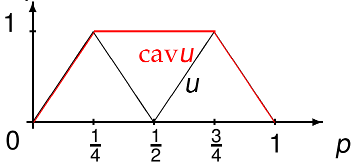

The value is $\mathrm{cav}\,u(p) = \inf\{f(p), f \text{ concave}, \ f \geq u\}$

$$= \max\left\{\sum_{m \in K} \lambda_m u(p_m), \forall m\ \lambda_m \geq 0, p_m \in \Delta(K), \sum_m \lambda_m = 1, \sum_m \lambda_m p_m = p\right\}.$$

(Aumann Maschler 1966, Kamenica Gentzkow 2011)

Sometimes not all splittings are available

Example: $K = \{0, 1\}$. The player (pharmaceutical firm) can run one or several experiments, each experiment returns the true state with proba $2/3$. Assume $p_0 = 1/2$ and $u(p) = \mathbf{1}_{p \geq 4/5}$ for all p.

An experiment from p_0 induces the distribution $\frac{1}{2}\delta_{2/3} + \frac{1}{2}\delta_{1/3}$ on $\Delta(K)$. From p, it induces

$$\varphi(p) = \frac{(1+p)}{3}\delta_{2p/(1+p)} + \frac{(2-p)}{3}\delta_{p/(2-p)}.$$

Optimum: run several experiments, stop if the current belief is $\geq 4/5$.

Say that f is S-concave if: $\forall p,\ f(p) \geq \frac{1}{3}(1+p)f(\frac{2p}{1+p}) + \frac{1}{3}(2-p)f(\frac{p}{2-p})$.

Theorem : the value is $v(p) = inf\{f(p), f\ S-\text{concave},\ \ f \geq u\}$

(*Fundamental theorem of Gambling*, Dubbins and Savage 1965)

Here $v(p_0) = 5/8$. v is not continuous on $[0, 1]$.

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We now study competition between 2 information designers, who control independent information.

Example: Competing firms release information about a new product to influence buyers' valuations.

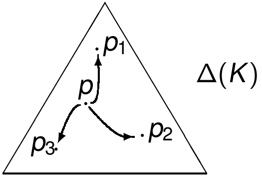
We use tools from the literature on dynamic games: Mertens-Zamir (1971), Heuer (1992), Renault-Venel (2017), Oliu-Barton (2018), Laraki-Renault (2020). Name reminiscent of "Long Cheap Talk" (Aumann Hart 2003).

Here : terminal payoffs and constraints on splittings.

Notations:

For $p \in \Delta(K)$, the set of all possible splittings of p is

$$SPL(p) = \left\{ s \in \Delta(\Delta(K)) : \int_{p' \in \Delta(K)} p' ds(p') = p \right\}.$$



Given $u : \Delta(K) \times \Delta(L) \to \mathbb{R}$ measurable bounded, we extend u to distributions by:

$$u(s,t) = \int_{(p,q) \in \Delta(K) \times \Delta(L)} u(p,q) ds(p) dt(q).$$

Model

- Finite set of unknown states $K \times L$, compact subsets P of $\Delta(K)$ and Q of $\Delta(L)$. Continuous payoff function $u : P \times Q \to \mathbb{R}$.
- Player 1 designs information about K, controls belief p in P. Available splittings at p: $S(p) \subset SPL(p) \cap \Delta(P)$, with $\delta_p \in S(p)$.

  Player 2 designs information about L, controls belief q in Q. Available splittings at q: $T(q) \subset SPL(q) \cap \Delta(Q)$, with $\delta_q \in T(q)$.

  S and T: continuous with convex compact values.
- At the end of each period, past splittings and posteriors are publicly observed.
- Finally, an agent takes an optimal decision as a function of his final posterior beliefs (p, q), inducing an expected payoff $u(p, q)$ for player 1 and $-u(p, q)$ for player 2.

Timing: we consider several game forms.

- T-period simultaneous game G_T and infinite horizon game G_∞: In each period t, given the current beliefs (p_{t-1}, q_{t-1}), simultaneously player 1 chooses a splitting in $S(p_{t-1})$ and player 2 chooses a splitting in $T(q_{t-1})$.
- T-period sequential games H_T and infinite horizon game H_∞: Player 1 splits in odd periods, player 2 in even periods.

Examples of admissible splittings:

- Standard case: player 1 can choose any experiment $x : K \to \Delta(M)$. Then $S(p) = SPL(p)$ for all p.
- More generally: player 1 has access to all experiments in a given compact subset X of $\{x : K \to \Delta(M)\}$.
- Player 1 can choose experiments with posteriors in a finite set A.

S-concavification and T-convexification

Let $f : P \times Q \to \mathbb{R}$.

Say that f is S-concave if for all p, q, $s \in S(p)$: $f(s, q) \leq f(p, q)$.
Say that f is T-convex if for all p, q, $t \in T(q)$: $f(p, t) \geq f(p, q)$.

$$\mathrm{cav} f(p, q) := \inf\{g(p, q), g(., q) \text{ concave}, g \geq f\},$$

$$\mathrm{vex} f(p, q) := \sup\{g(p, q), g(p, .) \text{ convex}, g \leq f\}.$$

Assumption 1 on available splittings:

Any distribution which can be reached in 2 stages can be reached in a single stage: $S^2 = S$ and $T^2 = T$.

Then for f continuous:

$$\mathrm{cav} f(p, q) = \max\{f(s, q), s \in S(p)\},$$
$$\mathrm{vex} f(p, q) = \min\{f(p, t), t \in T(q)\}.$$

Games with deadline

Proposition 1: Under A1,

Finite Sequential Games:
The game H_1 has value $\mathrm{cav}u(p_0, q_0) = \max_{s \in SP(p_0)} u(s, q_0)$,
The game H_2 has value $\mathrm{cav}\ \mathrm{vex}u(p_0, q_0)$ (same for all games H_{2T}),
The game H_3 has value $\mathrm{vex}\ \mathrm{cav}u(p_0, q_0)$ (same for all games H_{2T+1}),

Finite Simultaneous Games:
G_1 has value

$$OSV(p_0, q_0) = \max_{s \in S(p_0)} \min_{t \in S(q_0)} u(s, t) = \min_{t \in S(q_0)} \max_{s \in S(p_0)} u(s, t)$$

One-shot splitting value (Laraki, Sorin 2001 for the unconstraint case), may not exist if u is discontinuous. Same value for all games G_T.

Remark: The values (except for H_1) are all concave-convex. *It is always safe to remain silent if you can transmit information later.*

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Games without deadline

We use $\|.\|_1$ on P and $\|.\|_\infty$ on $\mathcal{C}(P,\mathbb{R})$.
Let us introduce the following set (from R-Venel 2017) :

$$D = \{f : P \to [-1,1], \forall p, p' \in P, \forall a, b \geq 0, |af(p) - bf(p')| \leq \|ap - bp'\|\}.$$

If $f \in D$, f is 1-Lispchitz.

D is a convex lattice, containing all constants in $[-1,1]$ and linear forms on $\mathbb{R}^K$ with norm ≤ 1.

Lemma The set $L :=_{def} \{\alpha f, \alpha \geq 0, f \in D\}$ is dense in $\mathcal{C}(P,\mathbb{R})$.

Notice that L contains all non revealing functions à la Aumann Maschler.

Assumption A2 on available splittings: $\forall f \in D, \mathrm{cav} f \in D$
(and similarly for player 2)

A2 is equivalent to: $\forall p, p' \in P, \forall s \in S(p), \forall a, b \geq 0,$

$$\exists s' \in S(p')\ s.t.\ \forall f \in D, |af(s) - bf(s')| \leq \|ap - bp'\|,$$

and similarly for T.

(It implies that $S : P \rightrightarrows \Delta(P)$ is non expansive for the distance $d^*(u, v) = \sup_{f \in D} |u(f) - v(f)|$. Close to a condition in R-Venel 17).

Illustration: Fix a single experiment $x : K \to \Delta(M)$. $p \in \Delta(K)$ induces the distribution:

$$\varphi(p) = \sum_{m \in M} x(p)(m)\delta_{l(p,m)},$$

with $x(p)(m) = \sum_k p^k x^k(m)$ probability of m, and $l(p,m) = (\frac{p^k x^k(m)}{x(p)(m)})_k$ conditionnal probability on K given m.

For f in D, p, p' in P, a, $b \geq 0$,

$$\begin{aligned} a\, f(\varphi(p)) - b\, f(\varphi(p')) &= \sum_{m \in M} a\, x(p)(m) f(l(p,m)) - b\, x(p')(m) f(l(p',m)), \\ &\leq \sum_{m \in M} \|a\,(p^k x^k(m))_k - b\,(p'^k x^k(m))_k\|, \\ &= \sum_{m \in M} \sum_{k \in K} |a\, p^k x^k(m) - b\, p'^k x^k(m)| = \|ap - bp'\|. \end{aligned}$$

and A2 is satisfied.
(can be extended to M compact)
(in general we don't have here: f 1-Lipschitz $\Longrightarrow \mathrm{cav} f$ is 1-Lipschitz)

Example: Assume player 1 has access to all experiments in a given compact subset X of $\{x : K \to \Delta(M)\}$ including the non revealing experiment.

For any "independent" strategy $\sigma = (\sigma_t)_{t\geq 1}$ with σ_t mesurable from $(X \times M)^{t-1}$ to $\Delta(X)$, and $T \geq 1$, denote by $\mu_T(p, \sigma)$ the induced law of the posteriors if σ is played up to T.

Define $S_T(p) = \{\mu_T(p, \sigma), \sigma \text{ varies}\}$ and finally $S(p)$ as the closure of $\cup_{t\geq 1} S_T(p)$.

S satisfies the assumptions $A1$ and $A2$.

Example: Assume P, Q finite. $A1$ and $A2$ are satisfied (up to a small modification in the definition of D).

Theorem: Under A1 and A2,

1) There exists a unique continuous function $v : P \times Q \to \mathbb{R}$ which is S-concave, T-convex, and such that for all $(p, q) \in P \times Q$:

$Q1$: if $v(p,q) > u(p,q)$, there exists $s \in S(p)$ s.t. $v(p,q) = v(s,q)$ and $v(p',q) \leq u(p',q)$ for all $p' \in \text{supp}\,(s)$,

$Q2$: if $v(p,q) < u(p,q)$, there exists $t \in T(q)$ s.t. $v(p,q) = v(p,t)$ and $v(p,q') \geq u(p,q')$ for all $q' \in \text{supp}\,(t)$.

v is characterized by $v = \text{cav}_S \min(u,v) = \text{vex}_T \max(u,v)$.

We write $v = MZ(u)$, the Mertens-Zamir value induced by u.

2) Both infinite horizon games G_∞ and H_∞ have pure stationary optimal strategies, and value: $MZ(u)(p_0, q_0)$.

Remark : for G_∞, H_∞, same as if the game stops after 2 silent moves.

Elements of Proof:
1) Uniqueness of $MZ(u)$: as in Laraki-R 2020.

Existence of $MZ(u)$: Let D_2 be the set of f in $\mathcal{C}(P \times Q)$ such that
$\forall p, p' \in P, \forall q, q' \in Q, \forall a, b \geq 0,$
$|af(p,q) - bf(p',q)| \leq \|ap - bp'\| \;\; and \;\; |af(p,q) - bf(p,q')| \leq \|aq - bq'\|.$

a) First assume that $u \in D_2$.

a1) Show that for each $\lambda \in (0,1]$, the value v_λ of the λ discounted game is in D_2.

a2) Consider a limit point $v = \lim_n v_{\lambda_n}$ and show that v is S-concave, T convex and satisfies $Q1$ and $Q2$. So $v = MZ(u)$. (close to Laraki-R 2020)

b) Conclude by density of $\{\alpha f, \alpha \geq 0, f \in D_2\}$ and the fact that $(u \mapsto MZ(u))$ is non expansive on D_2.

2) In the game G_∞ or H_∞, consider the strategy of player 1:
At (p, q), play $s \in S(p)$ given by $Q1$ (do nothing if $u(p, q) \geq v(p, q)$).
Consider any strategy of player 2.

$(p_n, q_n)_n$ is a bounded martingale, hence CV a.s. to some (p_∞, q_∞).
We want to show that $\mathbb{E}(u(p_\infty, q_\infty)) \geq v(p_0, q_0)$.

For $n \geq 0$, $\mathbb{E}(v(p_{n+1}, q_n)|(p_n, q_n)) = v(p_n, q_n)$ by $(Q1)$,

and $\mathbb{E}(v(p_{n+1}, q_{n+1})|(p_{n+1}, q_n)) \geq v(p_{n+1}, q_n)$ since v is T-convex.

So

$$\mathbb{E}(v(p_{n+1}, q_{n+1})) \geq \mathbb{E}(v(p_{n+1}, q_n)) = \mathbb{E}(v(p_n, q_n)) \geq v(p_0, q_0).$$

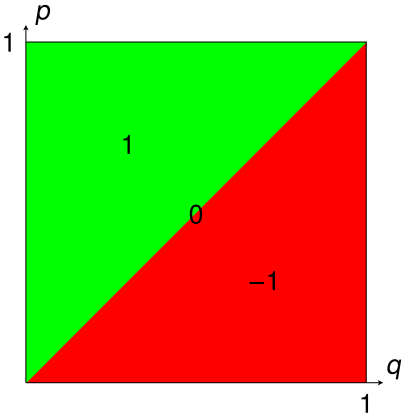
By $(Q1)$ again, $(p_{n+1}, q_n) \in \{u \geq v\}$ for each n, so passing to the limit we obtain $u(p_\infty, q_\infty) \geq v(p_\infty, q_\infty)$ a.s.

$$\begin{aligned}\mathbb{E}(u(p_\infty, q_\infty)) \geq \mathbb{E}(v(p_\infty, q_\infty)) &= \mathbb{E}(\lim_n(v(p_n, q_n))) \\ &= \lim_n \mathbb{E}(v(p_n, q_n)) \geq v(p_0, q_0).\end{aligned}$$

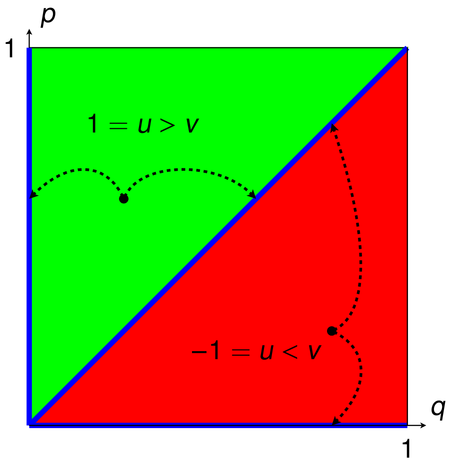
Remark: Generalized to a specific case of discontinuous u.

Illustration 1: being perceived as better

$K = L = \{0, 1\}$, all splittings are available. $p_0 = q_0 = 1/2$.
Discontinuous u function:



One-shot splitting value (Boleslavsky and Cotton, 2015) :
$OSV(p_0, q_0) = 0$ and it is optimal to to split uniformly on $[0, 1]$.



$\mathbf{u = v}$

$$v(p,q) = \frac{p-q}{\max(p,q)}$$

At equilibrium, the martingale moves at most once.

Illustration 2: Splitting on a finite grid

Fix P and Q finite subsets of $\Delta(K) \times \Delta(L)$. Assume all splittings on P and Q are available.

Example: $K = L = \{0, 1\}$, $P = Q = \{0, 1/3, 2/3, 1\}$ and

$u =$

$p = 1$	6	4	2	1
$p = 2/3$	4	2	4	3
$p = 1/3$	2	4	2	5
$p = 0$	0	2	4	7
	$q = 0$	$q = 1/3$	$q = 2/3$	$q = 1$

$$U = \begin{pmatrix} 6 & 4 & 2 & 1 \\ 4 & 2 & 4 & 3 \\ 2 & 4 & 2 & 5 \\ 0 & 2 & 4 & 7 \end{pmatrix}$$

$A = (A_{i,j})$ is S-concave if: $\forall j, \forall i \in \{2,3\}, a_{i,j} \geq 1/2 a_{i-1,j} + 1/2 a_{i+1,j}$.
$A = (A_{i,j})$ is T-convex if: $\forall i, \forall j \in \{2,3\}, a_{i,j} \leq 1/2 a_{i,j-1} + 1/2 a_{i,j+1}$.

$$\mathrm{cav} U = \begin{pmatrix} 6 & 4 & 2 & 1 \\ 4 & 4 & 4 & 3 \\ 2 & 4 & 4 & 5 \\ 0 & 2 & 4 & 7 \end{pmatrix}, \quad \mathrm{vex}\,\mathrm{cav} U = \begin{pmatrix} 6 & 4 & 2 & 1 \\ 4 & 11/3 & 10/3 & 3 \\ 2 & 9/3 & 12/3 & 5 \\ 0 & 2 & 4 & 7 \end{pmatrix}$$

$$\mathrm{vex} U = \begin{pmatrix} 6 & 4 & 2 & 1 \\ 4 & 2 & 5/2 & 3 \\ 2 & 2 & 2 & 5 \\ 0 & 2 & 4 & 7 \end{pmatrix}, \quad \mathrm{cav}\,\mathrm{vex} U = \begin{pmatrix} 6 & 4 & 2 & 1 \\ 4 & 10/3 & 8/3 & 3 \\ 2 & 8/3 & 10/3 & 5 \\ 0 & 2 & 4 & 7 \end{pmatrix}$$

How to compute $V = MZ(U)$, with $U = \begin{pmatrix} 6 & 4 & 2 & 1 \\ 4 & 2 & 4 & 3 \\ 2 & 4 & 2 & 5 \\ 0 & 2 & 4 & 7 \end{pmatrix}$?

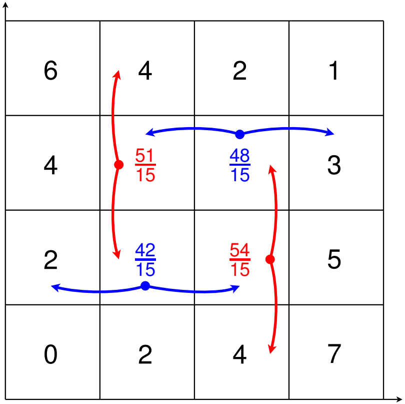
First check first and last rows and columns: OK, $V = \begin{pmatrix} 6 & 4 & 2 & 1 \\ 4 & a & b & 3 \\ 2 & c & d & 5 \\ 0 & 2 & 4 & 7 \end{pmatrix}$

Guess $a > 2$, $b < 4$, $c < 4$ and $d > 2$ (v>u, v<u, v=u).
Solve $a = (4 + c)/2$, $b = (a + 3)/2$, $c = (2 + d)/2$ and $d = (b + 4)/2$.
Then

$$V = \begin{pmatrix} 6 & 4 & 2 & 1 \\ 4 & 51/15 & 48/15 & 3 \\ 2 & 42/15 & 54/15 & 5 \\ 0 & 2 & 4 & 7 \end{pmatrix}$$

→ finite algorithm

Optimal strategies:



The number of disclosure periods is unbounded, but disclosure stops with probability 1 in finite time (reminiscent of the 4 frogs in Forges,1990).
(same phenomena can happen in the case of experiments.)

Approximation

Proposition 5: (Approximation) Consider the standard case with all splittings available and u continuous. *Assume binary states.*
For each $n \geq 1$, define U_n using uniform grids with stepsize $1/n$.
Compute V_n the MZ value of the matrix U_n.
Define $w_n : \Delta(K) \times \Delta(L)$ as the piecewise bilinear extension of V_n.

Then $(w_n)_n$ uniformly convergences to $MZ(u)$ when $n \to \infty$.

Splitting on a finite grid: general case

Assume P, Q finite and that non revealing splittings are available (no need for other assumption on S and T.
For all (p, q) in $A \times B$, define

$$v_-(p, q) = \sup\{v(s, q), \exists \alpha \in [0, 1), s \in \Delta(A \backslash \{p\}), \alpha\delta_p + (1-\alpha)s \in S(p)\},$$

$$v_+(p, q) = \inf\{v(p, t), \exists \alpha \in [0, 1), t \in \Delta(B \backslash \{q\}), \alpha\delta_q + (1-\alpha)t \in T(q)\}.$$

Proposition 6: (Long Information Design for general finite grids)
The games G_∞ and H_∞ have a value in pure strategies. This value is the unique function $v : A \times B \to \mathbb{R}$ which is S-concave, T-convex, and s.t. for all (p, q) in $A \times B$:

(Q1') if $v(p, q) > u(p, q)$, then $v(p, q) = v_-(p, q)$,
(Q2') if $v(p, q) < u(p, q)$, then $v(p, q) = v_+(p, q)$.

Remark: Pure ε-optimal strategies exist (use stopping times).

A few questions

Question 1: The conclusion of the theorem also hold by replacing $A2$ by $A'2$:
There exists a distance d on P such that: $\forall f$ 1-Lipschitz, $\mathrm{cav} f$ is 1-Lipschitz.
Fix P a convex compact subset of $\Delta(K)$ and assume all splittings with posteriors in P are available. Can we find d such that $A'2$ holds ?

Question 2: All splittings available on $P = \Delta(K)$, but u is severely discontinuous: $u(p, q) = 1$ if $p - q$ is rational, $= -1$ otherwise.
Both $v = 1$ and $v = -1$ are S-concave, T-convex and satisfy $(Q1)$ and $(Q2)$.
What about the value of G_∞ ?

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