SHORT TERM SCIENTIFIC MISSION (STSM) SCIENTIFIC REPORT

This report is submitted for approval by the STSM applicant to the STSM coordinator

Action number: CA16228
STSM title: Visiting Professor Sébastien Gerchinovitz for starting a research collaboration
STSM start and end date: 26/03/2019 **to** 26/04/2019
Grantee name: Tommaso Cesari

PURPOSE OF THE STSM (max 200 words)

The purpose of this STSM was to visit professor Sébastien Gerchinovitz at Université Toulouse III – Paul Sabatier to start a collaboration on active learning for level set identification.

In this setting the learner is interested in determining an approximation of the intersection between a sublevel and a superlevel of a given function with prescribed accuracy and the minimal amount of queries. More precisely, for any given real function f over a compact subset of the d-dimensional euclidean space, any real numbers a < b, and any non-negative precision parameter p, the goal is to output a set which includes the preimage of the compact interval [a, b] under f and it is included in the preimage of [a-p, b+p] under f with as few evaluations of f as possible.

A relevant special case is when a and b are both equal to the maximum of f. We planned to first study this case and then try to extend the techniques developed during this stage to the general problem.

DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS (max 500 words)

I began my work by studying how known algorithms find (approximations of) the set of all maximizers of a given function with the smallest amount of queries of the values of the function. Equivalently, this entails approximating the level set of all points in which a function attains their maxima. As mentioned above, this is a special case of the level set identification problem of particular importance. I joined an existing project which included my host, Sébastien Gerchinovitz, and Clément Bouttier. They had already performed a thorough review of the literature, and focused on the well-known Piyavski–Shubert algorithm. The algorithm works by querying values of the input function f sequentially, maintaining a proxy of f that bounds it from above under some suitable regularity assumptions. The performance of the algorithm are measured in terms of the simple regret, i.e., the difference between the maximum value of the function and the value of the function at the point returned by the algorithm. We investigated the possibility of using new techniques to improve upon previously known upper bounds on the simple regret of the Piyavski–Shubert algorithm for d-dimensional domains and how these techniques could be applied to the general level set identification problem. When I joined in, Sébastien and Clément had already proven tighter upper bounds for the deterministic case, i.e., when the queried values of the function can be observed exactly. I studied possible ways of simplifying this new analysis and subsequently worked on applying it to a more general setting where the queried values of the function are observed up to either deterministic adversarial perturbations, or stochastic subgaussian noise.

Insights from the fields of game theory, optimization, statistics, and machine learning (specifically, bandits) were useful to aid our investigations.

This part of the research ended up occupying a large part of my stay in Toulouse. The results we obtained (see below) will be put together in a stand-alone paper that we plan to submit to the Mathematics of Operations Research (MOR) journal by mid-summer 2019.

The study of the special case of the level set of the optimizers gave us many clues on how to proceed with the general level set identification problem. In particular, the 2018 paper "Adaptivity to Smoothness in X-armed bandits" by Andrea Locatelli and Alexandra Carpentier was key to pin down the right techniques for the task at hand. We were able to extend some of the ideas used in the new analysis of the Piyavski–Shubert algorithm as well as some of the techniques present in the Locatelli–Carpentier paper to the harder problems of identifying unions over intervals of level sets.

Some things remain to be understood and we plan on organizing a second, shorter visit this summer (see below) to complete the work. These results will be part of a separate paper which will include professor François Bachoc, from the Université Toulouse III – Paul Sabatier, as a third co-author.

DESCRIPTION OF THE MAIN RESULTS OBTAINED

Regarding the level set identification problem for the maximizers of a function f, we proved the following results for the Piyavski–Shubert algorithm.
Let f be a real function defined on any bounded subset of the d-dimensional Euclidean space. Assume that f attains its maximum at some point x and it is Liptschitz around x (this hypothesis, sometimes referred to as local smoothness, is far weaker than the usual Lipshitz assumption as it does not even implies that f is continuous anywhere but at x). We say that the simple regret after querying the function n times is the difference between the maximum value of the function and the value of the function evaluated at the point returned by the algorithm after n queries. We show that it is possible to upper-bound the simple regret of the Piyavski–Shubert algorithm if f is Lipschitz around one of its maxima. We bound the sample complexity (i.e., the minimal amount of queries needed to achieve a regret smaller than a given constant) by a quantity depending on the packing number of some sets. We then show a way to upper bound these packing numbers in terms of the prescribed accuracy for the regret, using a notion similar to the near-optimality dimension. We discuss how the Piyavski–Shubert is robust to small adversarial perturbations of the queried values and we leverage this robustness to prove analogous bounds in the stochastic setting, where values are observed up to subgaussian noise. We also investigated versions of the Piyavski–Shubert algorithm that are given an accuracy parameter and stop automatically guaranteeing that the value of f at the point returned by the algorithm differs from the maximum of the function by at most the accuracy. We show that this automatically-stopping variants are also robust to small perturbations and as such can similarly be applied to the stochastic setting.

The insights gathered during the first part of the mission gave us enough to design an algorithm to determine an approximating set for the general level set identification problem. The preliminary results that we have scale exponentially in the dimension d of the domain. We conjecture that however costly, such results cannot be improved but we did not prove a lower bound on the sample complexity yet. We leave this for a future visit (see below).

FUTURE COLLABORATIONS

During this mission we found some positive results for the general level set identification problem but some work still needs to be done. E.g., we are currently missing a negative result showing that our bounds are unimprovable. We plan on addressing this, and more questions by late July 2019. In my next visit, professor François Bachoc, who I had the pleasure to meet while in Toulouse, will join Sébastien Gerchinovitz and I.