SHORT TERM SCIENTIFIC MISSION (STSM)
SCIENTIFIC REPORT

This report is submitted for approval by the STSM applicant to the STSM coordinator

Action number: CA16228 Game Theory
STSM title: Computation of the disruption nucleolus of balanced games
STSM start and end date: 17/06/2019 to 22/06/2019
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PURPOSE OF THE STSM:

The main purpose of the visit was to investigate the simplification possibilities in the sequential linear programming computational scheme of the disruption nucleolus (Littlechild and Vaidya, 1976) in certain special classes of balanced cooperative games. The applicant recently identified conditions for balanced games under which the computation of various weighted nucleoli can be simplified (Solymosi, IJGT, 2019). The insight gained during that work coupled with the expertise of the host in the computation of the Gately (1974) allocation, a kind of special case of the disruption nucleolus (Staudacher and Anwander, 2019) was assumed to be particularly helpful in the proposed investigation.
Another purpose of the visit was to explore the possibilities of extending the publicly available R-package named CoopGame (https://cran.r-project.org/web/packages/CoopGame/index.html) developed by the host and his colleagues at Kempten, regarding specifically the computation of weighted nucleoli directly from the problem parameters without explicitly generating the coalitional function values.

DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS

During the visit, we have had daily meetings at Kempten University of Applied Sciences (typically from 9:30 to 17:00). First we have discussed what is known about the disruption nucleolus and its computation. It turns out that although in an experiment the disruption nucleolus is found to have a significantly better predictive power than the standard nucleolus (Michener et al., IJGT, 1981), this solution concept is mostly neglected in the literature. Then we have reviewed the transformation of the lexicographic nonlinear optimization procedure leading to the disruptive nucleolus to the standard sequential linear programming procedure of a specifically weighted nucleolus, and tried to identify special classes of balanced games where the known simplification possibilities could be applied. Assignment games were our first candidate. With the help of the CoopGame R-package we solved various examples, and concluded that we must focus on the subclass of exact assignment games.
On June 16, I gave an one-hour presentation on the applicability of Cooperative Game Theory in bankruptcy and liability situations under the title „On the Shapley value of constant-sum games and liability games".

DESCRIPTION OF THE MAIN RESULTS OBTAINED

Based on our explorative examples, we set up the conjecture that in the sequential linear programming procedure that can be used to compute the disruption nucleolus of an exact assignment game, only the single-player and the mixed-pair (consisting of one seller and one buyer) coalitions are needed, all other coalitions can be ignored. Interestingly, precisely this same family of quadratically many coalitions suffices

to determine the values of the exponentially many other coalitions and also the standard nucleolus even in non-exact assignment games (Solymosi and Raghavan, 1994). By means of counter-examples we demonstrated that the exactness condition cannot be lifted. The necessity of this restriction, however, is in accordance with an earlier observation on the dual games of assignment games (Nunez and Solymosi, 2017), namely that although the core of any assignment game is already determined by the dual values (the marginal contributions) of the single-player and the mixed-pair coalitions, this family suffices to generate the entire dual game only when the assignment game is exact. And in the sequential linear programming procedure the weights are precisely the differences of the dual and the original values of the coalitions. Naturally, proving our conjecture that would enable the polynomial time computation of the disruption nucleolus directly from the parameters of the underlying two-sided matching situation requires further theoretical investigations.

We have also identified another promising candidate for further study. Peer-group games (Branzei, Fragnelli, Tijs, 2002; Branzei, Solymosi, Tijs, 2005) seem to have the same feature as exact assignment games, namely that the difference of the dual and the original game is additively determined by a polynomial number of basic coalitions.

FUTURE COLLABORATIONS (if applicable)

We intend to continue the above project on the polynomial time computability of the disruption nucleolus, establish the conjectured results on the aformentioned classes of balanced profit games. We will present our findings in a joint publication. We also intend to extend the project to cost allocation games. Our primary candidates will be the fixed tree maintenance cost games, for these also seem to have the appropriate structure for the analogous simplifications than what we have identified for the aforementioned profit games.