SHORT TERM SCIENTIFIC MISSION (STSM) SCIENTIFIC REPORT

This report is submitted for approval by the STSM applicant to the STSM coordinator

Action number: CA16228 - European Network for Game Theory
STSM title: Stability in matching problems
STSM start and end date: 08/05/2019 to 05/06/2019
Grantee name: Ata Atay

PURPOSE OF THE STSM:

(max.200 words)

The purpose of this STSM was to begin a new collaboration with Prof. Vincent Vannetelbosch from Center for Operations Research & Econometrics (CORE) of the Université catholique de Louvain (UCLouvain). The main objective was twofold. First, we focus on solution concepts for the marriage problem (one-to-one matching problems). In this setting, we looked for conditions (probably related with the men-optimal and women-optimal stable matching) that characterize when the core (the set of stable matchings) of a marriage problem is a von Neumann-Morgenstern stable set. The second objective was to study a weaker notion of stability for the roommate problems which would always exist differently from stable matchings. Moreover, since it would based on the stability notion, our objective was to investigate its relationship with other solution concepts, such as the core and the bargaining set.

DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS

(max.500 words)

During my visit, we have had meetings with Prof. Vincent Vannetelbosch. Moreover, Prof. Ana Mauleon from UCLouvain joined our meetings.

In the first two weeks, we focus on the marriage problem. Our aim was to provide a characterization of the core stability by means of certain subproblems which respect the initial preference profile. To do so, we look for properties based on men-optimal and women-optimal matching. Based on an efficiency property, we wanted to introduce a notion of compatible subproblems with respect side optimal matchings. A similar approach has been considered for the analogous problem of assignment games. Nevertheless, in the marriage problem we have seen that defining such compatible subproblems are not possible due to the structure of the marriage problem.

Although we have seen that the first aim of the STSM does not lead to fruitful results, the second objective has been widely studied. We studied stability notions for the roommate problem. Since there may not exist a stable matching, we extended the notion of weak stability introduced for the marriage problem by Klijn and Massó (2003) to the roommate problem. We investigated the existence and the structure of weakly stable matcings and its relationship with a bargaining set.

Moreover, on May 24th I gave a talk titled "Multi-sided assignment games on m-partite graphs" at a workshop organized by Prof. Ana Mauleon at the Center for Research in Economics (CEREC), Université Saint-Louis.Bruxelles. On May 29th I presented a work in progress titled 'On the core of many-to-many assignment games' at the Brown Bag Seminar organized at the the Center for Operations Research & Econometrics (CORE), UCLouvain.

DESCRIPTION OF THE MAIN RESULTS OBTAINED

First, we study the marriage problem and the possibility of a characterization of the core stability. That is the say, when the set of stable matchings coincide with the von Neumann-Morgenstern stable set. We observed that do not the structure of the marriage problem, such a characterization cannot be obtained by means of certain subproblems as it has been obtained for the two-sided assignment games.

In the second part, we investigate the roommate problem. Differently from the marriage problem, in roommate problems stable matchings may not exist. During this STSM, instead of restricting ourselves to the domain of roommate problems in which stable matchings exist, we propose a weaker notion of stability. Our main aim was to propose a solution based on the stability notion is guaranteed for the unsolvable roommate problems (problems without stable matchings) as well. In the case of the marriage problem, Klijn and Massó (2003) introduce the so-called weakly stable matchings. A matching is said to be weakly stable if a partner can find a more preferred blocking partner than the current one with whom she can constitute another blocking pair. We investigate the existence and the structure of weakly stable matcings and its relationship with a bargaining set.

Stable matchings, by defintion, are weakly stable matchings. Hence, we guarantee the existence of weakly stable matchings for problems with the non-empty set of stable matchings. First, by means of constructing a matching for any problem without stable matching, the existence of weakly stable matchings for the roommate problem is guaranteed. Moreover, we introduce a procedure that returns a weakly stable matching. Therefore, we have seen that, in the roommate problem, unlike the set of stable matchings, the set of weakly stable matchings is always non-empty.

After the existence of weakly stable matchings is guaranteed, we provided examples to show their relationship with the stable matchings. We have provided a roommate problem with an empty core in which there exists a weakly stable matching. Also, another example of a solvable roommate problem in which there are more weakly stable matchings than stable matchings was given. Hence, we see that, whenever the set of stable matchings exists, it is a (strict) subset of the weakly stable matchings.

Then, we address another objective of this STSM. Namely, the relationship between the set of weakly stable matchings and the bargaining set of Zhou (1994). We adapt the variation of bargaining sets introduced by Zhou (1994) to the roommate problem. We first show that weak stability is not sufficient for a matching to be in the bargaining set of Zhou (1994). Even though weak stability does not guarantee the existence of the bargaining set, we construct a matching that is always in the bargaining set of a roommate problem. We have shown that the bargaining set for a roommate problem is always non-empty. Since the core of a roommate problem may be empty, we have provided another set-wise solution concept which is always non-empty and contains the core whenever it is non-empty.

Finally, we generalize the characterization result of Klijn and Massó (2003) to the roommate problem: the bargaining set of Zhou (1994) coincides with the set of weakly stable and weakly efficient matchings.

FUTURE COLLABORATIONS (if applicable)

We will now write a working paper following the results that we obtain. We plan to have a working paper in the following months. Moreover, we have discussed several new questions on the solution concepts introduced for matching problems. We hope to work on questions that we have posed during my STSM at the Center for Operations Research & Econometrics (CORE) of the Université catholique de Louvain (UCLouvain). I sincerely thank the COST Action for their support which made this visit possible.