SHORT TERM SCIENTIFIC MISSION (STSM) – SCIENTIFIC REPORT

The STSM applicant submits this report for approval to the STSM coordinator

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PURPOSE OF THE STSM/

The purpose of this visit was to begin a collaboration with Prof. Tamas Solymosi from Corvinus University of Budapest and with Ata Atay, postdoc at the Hungarian Academy of Sciences. This new project is based on our previous works, both individually and together, on bilateral assignment markets where each buyer can buy at most one unit from one of the sellers. We now intend to analyse bilateral markets where multiple partnership is allowed, that is, each seller owns several items and each buyer may buy several items, even more than one from the same seller. The core of these market games is known to be non-empty (Sánchez-Soriano, 2001; Sotomayor, 2002), but little is known about the structure of the core and properties of core allocation rules.

DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS

During my visit, we have had daily meetings at Corvinus University (tipically from 10:00 to 18:00 or 10:00 to 16:00) to discuss first what is known about the core of these many-to-many markets with transferable utility, and to pose open questions. We developed examples of such markets, computed some single-valued core solutions for these examples, found counterexamples of some conjectures and obtained some results for the many-to-one case, which I describe below.

I also presented a paper in a seminar organized on October 11th, titled "Valuation monotonicity, fairness and stability in assignment problems" and attended another game theory seminar at Corvinus University on October 12th.

DESCRIPTION OF THE MAIN RESULTS OBTAINED

After reviewing the existing literature we focused our attention on the many-to-one case. That is, bilateral assignment markets where agents on one side can be matched with at most one agent on the other side, while agents on this other side can match several agents on the first side. A valuation matrix gathers the valuation of each mixed-pair of agents. This model may represent a job market where agents can work in

only one firm, while each firm may hire several workers, as many as the given capacity of the firm. For this model, the core is known to have a lattice structure (as in the one-to-one case), which guarantees the existence of an optimal core allocation for each sector (Sotomayor, 2002). However, the rule that assigns to each such market the firms-optimal core allocation loses some properties it had in the one-to-one case (for instance, strategy-proofness). This rule can now be manipulated.

Our first result is a characterization of the two-optimal core allocations in terms of a directed graph. From this graph we can extract an order and define a recursive procedure to obtain these two optimal core allocations. This procedure only takes into account the valuation matrix and, because of that, we think it will be useful to prove monotonicity properties of these two optimal rules.

We have also obtained some results related to other solution concepts for coalitional games. We have seen that, different to the one-to-one case, the bargaining set of the many-to-one assignment game may be greater than the core, and the kernel may not be included in the core. Also, the tau-value, that in one-to-one assignment games coincides with the midpoint of the buyers-optimal core allocation and the sellers-optimal core allocation, may lie outside the core in the many-to-one case.

FUTURE COLLABORATIONS (if applicable)

We will now write in a working paper the results obtained and continue investigating in order to complete a paper to be submitted to some international game-theory journal. We expect that the graph representation of the optimal core allocations and the related algorithm will allow to check whether these two allocation rules are pairwise monotonic and/or have some other monotonicity property (as those proved in the one-to-one case by van den Brink et al., 2018).

The fair-division point, which is the midpoint between the two optimal core allocations, will also be investigated.

Other questions, such as the existence of optimal core allocations when both sides of the market are allowed to establish multiple partnerships, are left for future collaboration.