

## SHORT TERM SCIENTIFIC MISSION (STSM) SCIENTIFIC REPORT

This report is submitted for approval by the STSM applicant to the STSM coordinator

**Action number:** CA16228 – European Network for Game Theory  
**STSM title:** Common Charges: A Finitely Additive Approach to Common Priors  
**STSM start and end date:** 27/09/2018 to 09/10/2018  
**Grantee name:** David Bartl

### PURPOSE OF THE STSM:

(max.200 words)

Samet (1998) characterised the common prior in finite knowledge belief spaces. Feinberg (2000) showed by a counterexample that Samet's characterization does not work in general, even for countable type spaces. Lehrer and Samet (2014) considered countable type spaces and provided three characterisations of common prior. One of Lehrer and Samet's characterisation says that weak trade consistency is equivalent to weak belief consistency.

In this project we consider a setting in which the players' beliefs are probability charges rather than probability measures, differing from Feinberg (2000) and Lehrer and Samet (2014), and we examine whether Samet's characterization works in this setting. Furthermore, we plan to show that a characterisation similar to Lehrer and Samet's holds for any type space.

### DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS

(max.500 words)

Tentatively, the research stay should have taken place from 28th Sept. to 5th Oct. 2018. Due to logistic reasons (including the teaching duties), the stay took place from 27th Sept. to 9th Oct. 2018, i.e., started earlier and ended later, in order to utilize the possibility of the joint work as much as possible.

It was our main propose to work on a manuscript on the topic. We could not finish the paper, but we have already attained the following results:

1. The appropriate model for beliefs in infinite games using charges has been detailed.
2. The characterisation of common prior by Samet (1998) can be generalized to the infinite case by charges.
3. Lehrer and Samet (2014)'s characterisation, in which weak trade consistency is equivalent to weak belief consistency, is a corollary of our charge type generalisation of Samet (1998)'s results.
4. The counterexample by Feinberg (2000) fails in our setting of charges.

### **DESCRIPTION OF THE MAIN RESULTS OBTAINED**

(max.500 words)

We have made progress in composing the manuscript. We have put together several lemmata and theorems, and in addition clarified many notions. We have characterised the common prior by non-agreeable bets if the beliefs are probability charges. It is possible to follow Samet (1999)'s results in this general setting, but – since we are considering general infinite type spaces – the steps call for non-trivial tools of mathematics.

In particular, we have constructed examples in which, contrary to the basic result of Aumann (1976), there coexist both common priors and agreeable bets. Careful analysis of these examples indicates that how the notion of 'prior' is defined becomes crucial. If one defines a posterior as following from a prior under a conditional probability then one result is attained. If one defines first the types and then asks, in a measure theoretic way, what could be the priors, a different result follows. These two definitions until now have been equivalent in the literature; we show now that they must be considered separate.

### **FUTURE COLLABORATIONS (if applicable)**

(max.500 words)

There are still many open problems to research, including technical and conceptual issues, alongside finding applications for our results. For example, one technical problem is the description of the set of priors of a player as the convex hull of the types. This is very elegant in the finite setting of Samet (1998). It is not clear yet whether a similar characterisation is possible in our infinite setting. We also plan to relate our results to the no-trade literature.

The main take-home message we have learned from the short-term scientific mission is how effective these missions are in conducting research. We have made great progress and we clarified many issues. Personal communication indeed makes research much more effective.

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