SHORT TERM SCIENTIFIC MISSION (STSM) – SCIENTIFIC REPORT

The STSM applicant submits this report for approval to the STSM coordinator

Action number: CA16228
STSM title: A Fine-Grained View of Matching Under Preferences
STSM start and end date: 20/05/2018 to 26/05/2018
Grantee name: Dr. Matthias Mnich

PURPOSE OF THE STSM

(max.500 words)

Problems of matching under preferences have been a core part of game theory since the development of the seminal Gale-Shapley algorithm in the 1960s. Not only have they led to theoretically beautiful algorithms, but also had a profound impact on questions of practical relevance; this has been recognized by the Nobel Prize in Economics in 2012. Traditional considerations of computational complexity of matching problems focussed on the P vs. NP- dichotomy. This restriction to coarse-grained polynomial-time vs. exponential-time complexities is surprising given the multitude of parameters and structural aspects which arise from practical applications: lengths of preference lists, distinctiveness of preference lists, people that must be matched, capacities of hospitals etc. are just some of these relevant aspects. We will work on a fine-grained complexity view of matching under preferences Mnich and Schlotter [2017], employing the tools of parameterized complexity. This way, we aim to develop a rich set of fixed-parameter algorithms for settings more general than existing polynomial-time algorithms, as well as identifying those parameters which are the source of a problem's intractability manifested by its NP-hardness. Our goal is to develop a set of fixed-parameter algorithms for classified stable matchings problem Huang [2010], which are among the most general matching problems which have been studied. Thereby we are advancing research on this important subject of game theory.

C.-C. Huang. Classified stable matching. In Proc. SODA 2010, pp. 1235–1253, 2010.

M. Mnich and I. Schlotter. Stable marriage with covering constraints–a complete computational trichotomy. In Proc. SAGT 2017, pp. 320–332, 2017.

DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS

(max.500 words)

First, with Dr. Péter Biró and Prof. Dr. Kolos Csaba Ágoston at Corvinus University, we discussed the use of integer programming methods for stable marriage problems. The motivation for this line of research was that integer programming methods are quite successful when it comes to solving stable matching in practise, be it for kidney exchange, student-course allocation, or other fields; and these two researchers are experts as witnessed by recent practical implementations [1]. On the other hand, recent advances by the grantee in the theory of integer programming [2] led to powerful multivariate algorithms for several scheduling problems and problems on strings, so explored the possibilities of deriving multivariate algorithms for stable matching problems using integer programming.

Second, with Dr. Ágnes Cseh at the Institute of Economics of the Hungarian Academy of Sciences we considered finding maximum-size stable matching problems with ties and incomplete lists. These problems have been well-investigated from the viewpoint of approximation algorithms, but despite this, their exact approximability is still not fully understood even in many special cases such as when all ties are only one-sided. We therefore discussed potential parameters for such problems, for which one could attempt to design fixed-parameter algorithms which could then be used as subroutines for approximation algorithms with better performance guarantees than those known in the literature. We further explored the power and limitations of linear programming methods for such problems.

[1] Kolos Csaba Ágoston, Péter Biró, Iain McBride: Integer programming methods for special college admissions problems, Journal of Combinatorial Optimization 32(4), pp. 1371–1399, 2016.

[2] Martin Koutecký, Dusan Knop, Matthias Mnich: Combinatorial n-fold integer programming and applications. Proc. ESA 2017, LIPIcs **87**, pp. 54:1–54:14, 2017.

DESCRIPTION OF THE MAIN RESULTS OBTAINED

(max. 500 words)

For the practical applications discussed, such as matching experts to projects, we realized that the notions of stable matchings and popular matchings are probably not well-funded. It appears that different though related concepts such as socially-stable matchings, and exchange-stable matchings, better capture the satisfication of people that are matched by such schemes. In that regime though, it is not evident though how to design integer programming models which exhibit nice combinatorial properties which can be exploited algorithmically. However, an interpretation of basic stable matching problems in terms of a scheduling problem was discovered, which yields fixed-parameter algorithms for very basic scenarios of stable matching through existing fixed-parameter algorithms for scheduling problems. It appears possible to extend them to more general cases as long as the agents' preference orders exhibit some nice structural properties.

For the problem of finding maximum-size stable stable matchings with incomplete lists, it is indeed possible to design multivariate algorithms based on integer programming methods, in the natural scenario that the number of people to matched can be classified into few classes. Such scenario happens for instance in the student-course allocation projects considered by Dr. Péter Biró. One result obtained is that one can design an algorithm for this problem with double-exponential dependence in the number of types; future research must show whether the run time can be improved to single-exponential using better tools.

FUTURE COLLABORATIONS (if applicable)

(max.500 words)

We decided that it is worth further exploring the work initiated during this week. In particular, it seems promising to design integer programming algorithms with provable theoretical guarantees, for two reasons: first, to understand the better the practical successes of integer programming for stable matching problems from a theoretical perspective; and second, to develop general-purpose tools which are applicable to a wide variety of matching scenarios, so that practitioners in student-course allocation, kidney exchange etc. do not need to redevelop the means already useful in other application domains but can resort to these tools.

Further, it seems very promising to design fixed-parameter approximation algorithms for stable matching problems. Several problems that we considered during this week are unlikely to admit algorithm yielding exact optimal solutions in fixed-parameter time modulo standard complexity-theoretic hypotheses, nor is their polynomial-time approximability well-understood. By interpolating between these two extremes and designing algorithm that yield approximate solutions in fixed-parameter time we aim to shed new light on some of these questions which have been open for several years now.