SHORT TERM SCIENTIFIC MISSION (STSM) – SCIENTIFIC REPORT

The STSM applicant submits this report for approval to the STSM coordinator

Action number:
STSM title: Congestion Games under Uncertainty
STSM start and end date: 21/01/2018 to 07/02/2018
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PURPOSE OF THE STSM

(max.500 words)

In a congestion game there is a set of available resources and the action space of each player is a subset of this set of resources (not necessarily the same subset). The cost of using each resource is a non-decreasing function of the number of players who use it. One of the main objects of interest in the literature about congestion games has been the efficiency of their equilibria. This has typically been measured using either the Price of Anarchy or the Price of Uncertainty. In order to compute these quantities it is obviously necessary to know exactly all the details of the game. In reality many aspects of a congestion game are unknown to the players. For instance the cost of using a resource may be affected by external random elements (think of routing games, where the travel time over a road depends on the weather conditions). Even the set of players may be random (every morning not all the workers go from home to their office). The set of resources and hence the set possible actions could also be random (one road could be closed because of work).

The goal of this research is to examine congestion games under uncertainty. First we will consider the simpler case of static games and then we will move to dynamic games, focusing on the case of network games, where players make a decision at every node that they reach in the network. We want to study existence of equilibria and, when they exist, their efficiency.

DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS

(max.500 words)

A. We have considered a simple model of congestion game with two resources. In the classical example of congestion games on networks, the resources are edges of a graph (in this specific case, a parallel graph). The probability that the first is functioning is a decreasing function h of the number of agents who want to use that resource. The other resource works with probability zero. This second resource can be seen as a waiting room. Agents arrive at the system with a fixed arrival rate and aim at being served—i.e., using one working resource and leaving—in the shortest possible time. This gives rise to a non-cooperative stochastic game.

The main issues that we have considered are:

1. Stability conditions for the system not to explode.
2. Existence and computation of Nash equilibria of the game.
3. Comparison of the equilibrium outcome of the game with the one that a benevolent planner would have obtained.

B. We have also laid the foundations for a different related problem of behavior of agents on a network in the presence of uncertainty. In this model heterogeneous consumers sequentially arrive at some market with the need to buy a product of unknown quality. When they arrive, they connect to a random subset of existing agents and they read the review of the consumers in this subset who have bought the product before. The main issue is whether social learning can be achieved, i.e., whether consumers asymptotically learn the unknown quality of the object and therefore can make the right purchasing decision. The problem can be modeled as a game of incomplete information. A more sophisticated version of the model includes the seller in the set of players and gives her the possibility of strategically choose the price of the product.

DESCRIPTION OF THE MAIN RESULTS OBTAINED

(max. 500 words)

A. With reference to the congestion game under uncertainty, we have provided some necessary conditions for the system to be stable in terms of the following two quantities: (a) the probability that the first resource is functioning, as a function h of the number of agents and (b) the arrival rate of customers. Then we have imposed some conditions on the function h and we have looked at existence of Nash equilibria. We have considered some techniques for the computation of their payoff. We have also solved an optimization problem where the choice of all agents is determined by a benevolent planner.

B. With reference to the learning game on networks, we have seen that a fully Bayesian update of beliefs is computationally intractable and have looked for a simpler non-Bayesian approach that allows each consumer to use the previous consumers' reviews to estimate the unknown quality of the product without knowing which approach they used for their own estimation. This procedure involves the method of moments and a version of the strong law of large numbers.

FUTURE COLLABORATIONS (if applicable)

(max.500 words)

A. We plan to extend the generality of the model, by considering larger classes of probability functions. We will then try to find conditions for the existence of equilibria and then we will consider their multiplicity and their efficiency, using tools like the Price of Anarchy or the Price of Stability. This part of the research will

involve also Jérôme Renault (Toulouse School of Economics) and Gaëtan Fournier (Université Aix-Marseille).

B. We want to consider a general statistical method for mixture models and to compare it in terms of efficiency with the fully Bayesian method. Then we want to see how this procedure can be applied to our model of social learning with review on social networks.